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# VISCO-HYPOPLASTIC MODEL FOR STRUCTURED SOILS

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## ABSTRACT

The Visco-hypoplastic constitutive model is developed for fine-grained soils and reproduces the viscous behavior of these materials such as creep, relaxation and rate dependence. The model adjusts to the Butterfield's compression law at isotropic states. Nevertheless, it is not able to simulate the influence of the structure in the materials mechanical behavior and hence, an extension has been proposed herein. This extension introduces a new state variable representing the material structure defined as the ratio of the preconsolidated mean stress at intact and remolded states for the same material. The structure follows a degradation law which accelerates after yielding and decreases until the remolded state is reached. The overconsolidation ratio OCR and the material stiffness increase along with the structure values. The structure influence in the stiffness is reproduced by modifying the Butterfield's compression law at the elastic range. Simulations on a high structured soil show satisfactory results, and a numerical implementation for FEM has been developed.

## INTRODUCTION

The present work proposes a modification of the Visco-hypoplastic model (Niemunis, A. and Krieg, S. 1996., Niemunis 1996) in order to reproduce structured soils. The Visco-hypoplastic model belongs to the hypoplastic framework and is developed for fine grained soils with normally or lightly overconsolidation ratios OCR. The constitutive model is able to reproduce viscous effects such as compression isotachs, creep and relaxation of stresses at constant volume. Nevertheless, it is not able to reproduce the influence of the structure on the mechanical behavior. The proposal presented herein extends the reference model in order to incorporate the effects of the structure adopting some existent expressions following (Baudet and Stallebrass, 2004; Rouiania and Wood 2000; Masin, 2006), but in constrast, some modifications are developed aiming to keep the capability of reproducing the viscous effects.

## REFERENCE VISCO-HYPOPLASTIC MODEL

### *Preliminaries*

The Visco-hypoplastic constitutive model is deduced from the Wollffersdorff hypoplastic version (Wolffersdorff, 1996). This model reproduces the mechanical behaviour of sands with the following constitutive equation,

$$\overset{\circ}{\mathbf{T}} = h(\mathbf{T}, \mathbf{D}, e) = f_s \mathcal{L}:\mathbf{D} + f_s f_d \mathbf{N} \|\mathbf{D}\| \qquad (1)$$

where, $\overset{\circ}{\mathbf{T}}$ is the Zaremba-Jaumann co-rotational stress rate tensor, $\mathbf{D}$ is the strain rate tensor, $\mathcal{L}$ is an hypoelastic fourth order stiffness tensor, $\mathbf{N}$ is a stiffness tensor non-linear in $\mathbf{D}$, $e$ is the void ratio and $f_s$, and $f_d$ are scalar factors (Kolymbas, 1991; Wolffersdorff, 1996; Bauer, 1996).

### *Constitutive Visco-hypoplastic model*

The Visco-hypoplastic model considers the Maxwell strain decomposition in an elastic and viscous part. In rate form, the strain decomposition gives,

$$\mathbf{D} = \mathbf{D}^{e} + \mathbf{D}^{vis} \tag{2}$$

where $\mathbf{D}^{e}$ is the elastic strain rate tensor and $\mathbf{D}^{vis}$ is the viscous strain rate tensor. The Visco-hypoplastic mathematical form is revealed when comparing equations (1) and (2):

$$\overset{\circ}{\mathbf{T}} = f_b \hat{L}:(\mathbf{D} - \mathbf{D}^{vis}) \tag{3}$$

where $\hat{L}$ is the hypoelastic stiffness tensor without scalar factors, $f_b$ is the barotropy factor which adjusts the constitutive equation to the Butterfield's compression law (Niemunis, 1996; Butterfield, 1979). The barotropy factor is adjusted to the elastic condition $\mathbf{D}^{vis} = 0$ and reads,

$$f_b = \frac{\mathrm{tr}\mathbf{T}}{(1 + a^2/3)\kappa} \tag{4}$$

with $\mathrm{tr}(\mathbf{T})$ being the trace of the Cauchy stress $\mathbf{T}$, $\kappa$ the swelling index for the Butterfield relation and $a = f(\varphi_c)$ a material constant which depends on the critical friction angle $\varphi_c$. The viscous strain rate tensor $\mathbf{D}^{vis}$ follows the Norton's law and keeps the direction of the hypoplastic flow rule $-\vec{\mathbf{B}}$ (Niemunis, 1996),

$$\mathbf{D}^{vis} = -\vec{\mathbf{B}} D_r \left( \frac{1}{\mathrm{OCR}} \right)^{1/I_v} \tag{5}$$

where OCR is the overconsolidation ratio, $I_v$ is the Leinenkugel's viscosity index and $D_r$ is the reference viscous strain rate. In order to define the OCR for all stresses states, Niemunis introduced the Modified Cam-Clay yielding surface. This surface is used to project any stress point on the isotropic plane denoted by $p_e^{+}$. The OCR is computed as the ratio of $p_e^{+}$ and the Hvorslev mean stress $p_e$ projected in a reference isotach (arbitrarily selected) with OCR=1, which presents a predefined void ratio $e_{100}$ for $p$=100 kPa.

## STRUCTURE INCORPORATION

### *Preliminaries*

Despite that the soil structure concept is commonly encountered in the literature, still a discussion in its definition. In a macromechanical point of view, the structure intends to explain the differences between the mechanical behaviour of a structured and remolded sample of the same material. Some experiemental evidence indicate a significant variation of the preconsolidation mean stress and the elastic stiffness for the same initial conditions (Lereueil and Vaughan, 1990; Burland, 1990).

These behaviour may be atributted to the soil fabric and the arrangement of particles (Mitchel, 1993). In addition, some cementation due to chemical processes may be acting in the inter-particle spaces (Lereueil and Vaughan, 1990; Mitchel, 1993).

Several constitutive models have been extended in order to incorporate the structure influence (Baudet and Stallebrass, 2004; Rouiania and Wood 2000; Masin, 2006, Nova et. al., 2001). Some of these models decompose the stress tensor in a re-molded and cemented part (Nova et. al., 2001). The sum of the two components gives the total stress:

$$\mathbf{T} = \mathbf{T}_B + \mathbf{T}_R \tag{6}$$

Under the consideration of this decomposition, the remolded part $\mathbf{T}_R$ is simulated by an existent constitutive model, whereas the cemented part $\mathbf{T}_B$ is simulated by a proposed model. Nevertheless, the definition of its initial conditions might be a complex task, especially when using the constitutive model within a finite element code. Nova (2001) suggested to find these initial conditions by doing $K_0$ path, which begins from the initial state free of stresses $\mathbf{T}_0 = 0$. This approximation may be qualified as unrealistic or even unacceptable if the goal is to reproduce the geological history. Under the consideration of these facts, the decomposition of stresses in order to reproduce the soil structure is disregarded in this work.

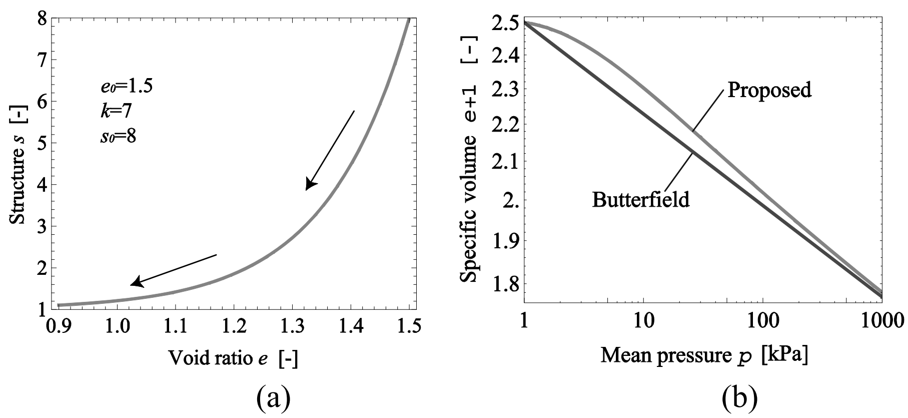



Figure 1: Example of the proposed compression law. a) exponential decay law of $s$ with void ratio $e$. b) proposed compression law.

A different approach (Rouiania and Wood 2000; Baudet and Stallebrass, 2004) considers the structure as a single state variable $s$, which follows a degradation law until a residual value is reached. This approach might be convenient for its incorporation in an existent model because of its simplicity. For example, Masin (2006) suggested the incorporation of $s$ as a factor in the Hvorslev mean stress $p_e = s p_e^*$, where the asterisk on $p_e^*$ denotes remolded states. For the present work, this equation is adopted as a formal definition of the structure.

A simple extension of the Visco-hypoplastic model that has incorporated this equation was presented in (Fuentes, 2009). Nevertheless, a modification to this model may be convenient in order to simulate the increase on the stiffness for $s > 1$ and the dependence of the structure degradation with the OCR, facts which have been experimetally observed (Lereueil, et. al., 1990).

***Extended Visco-hypoplastic model***

The proposed constitutive model incorporates the influence of the structure in the stiffness and developes a degradation law for $s$ which depends on the OCR. Taking into account that the barotropy factor is deduced for the elastic condition $\mathbf{D}^{vis}=0$, the modified Butterfield's compression law for swelling is proposed to be,

$$\ln\left(\frac{1+e}{1+e_0}\right)=(-\kappa+\Gamma\ln(s))\cdot\ln\left(\frac{p_e}{p_0}\right) \tag{7}$$

where $\Gamma$ is a material parameter which represents the part of the swelling index associated with the structure. For higher values of $s$ the proposal reproduces a decrease in the classical definition of $\kappa$ and recovers its original value when $s=1$, considered as the remolded state. An illustrative example is shown in Figure 1. The structure $s$ follows an exponential decay law with the void ratio (Figure 1.a). For this example, the new compression law shows an apparent high stiffness for $s=s_0$ and recovers the original Butterfield's compression law when $s=1$.

With Eq. (10), the modified barotropy factor reads,

$$f_b=\frac{3p}{(\kappa-\Gamma\ln(s))\cdot(1+a^2/3)} \tag{8}$$

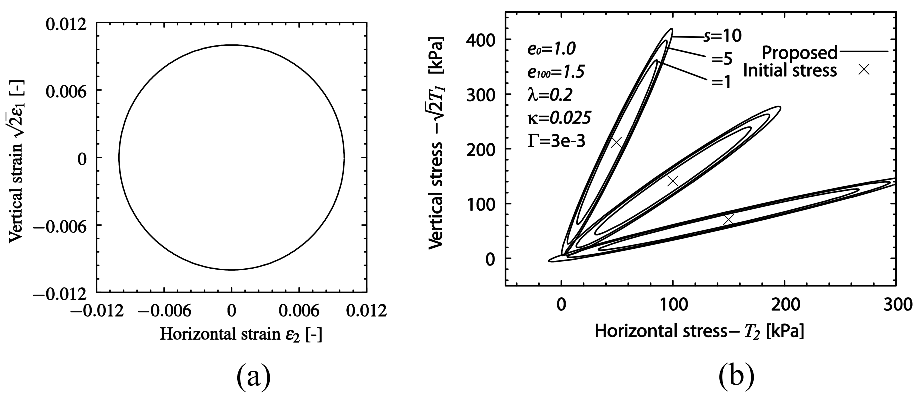



Figure 2: Responses envelopes for different initial structure values $s_0$.

and hence, an increment of the stiffness is reproduced for $s>1$. Figure 2.a-b show the simulation of responses envelopes (Gudehus, 1979) for different values of $s_0$. Results show greater responses envelopes for $s>1$, which is in accordance with the modified Butterfield's compression law. Figure 3.a-d show 3D responses envelopes, each with different initial structure values $s_0$.

According to Baudet and Stallebrass (2004) the structure follows an exponential decay law.

$$\dot{s}=-\frac{k}{\lambda}(s-1)\dot{\varepsilon}^{dam} \tag{9}$$

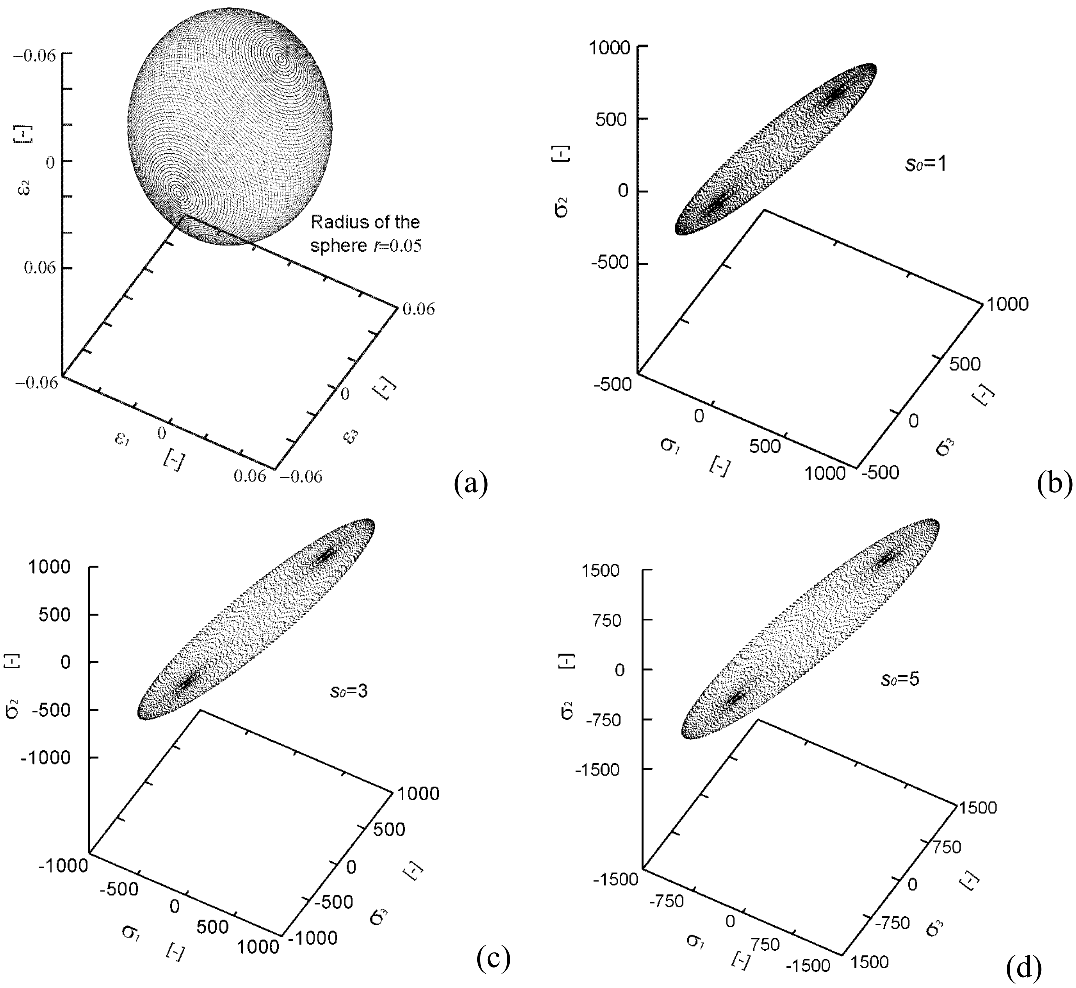



Figure 3: 3D Responses envelopes for different initial structure values. a) Strain sphere, b) response envelope for *s=1,* c) for *s=3*, and d) for *s=5.*

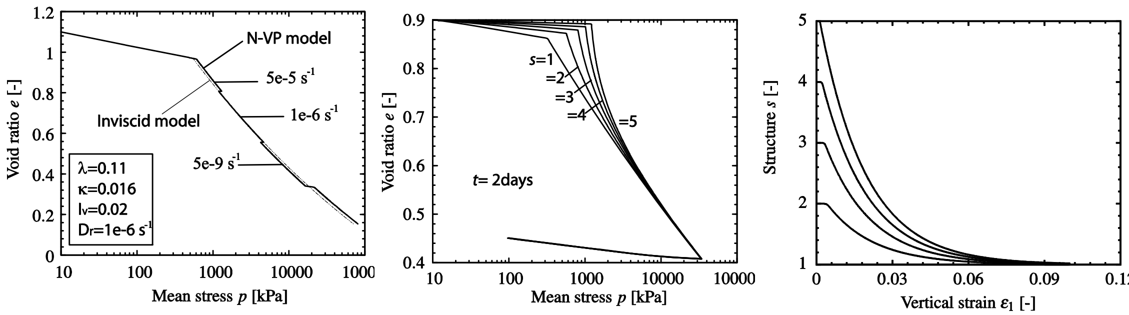
where $\dot{\varepsilon}^{dam}$ being the damage strain rate. Nevertheless, experimental evidence indicates an acceleration on the degradation of *s* after yielding. Subsequently, a suitable equation for the damage strain ratio $\dot{\varepsilon}^{dam}$ may be,

$$\dot{\varepsilon}^{dam} = \exp(\mathrm{OCR}^*)\sqrt{\dot{\varepsilon}^{v2} + \dot{\varepsilon}^{s2}} \tag{10}$$

where $\dot{\varepsilon}^{v} = \mathrm{tr}(\mathbf{D})$, $\dot{\varepsilon}^{s} = \sqrt{\frac{2}{3}}\left\|\mathbf{D}^*\right\|$ are the volumetric and deviatoric strain rate invariants respectively, and $\left\|\mathbf{D}^*\right\|$ is the norm of the deviatoric strain rate tensor. The asterisk on OCR$^*$ indicates that the viscosity has been disregarded in its definition (Under the Visco-hypoplastic framework, the OCR depends on the strain rate). In order to deduce an inviscid OCR, the current value is compared with the OCR deduced for the creep condition $\overset{\circ}{\mathbf{T}} = 0$. The inviscid OCR$^*$ reads,

$$\mathrm{OCR}^* = \left(\frac{\sqrt{3}D_r}{\left\|\mathbf{B}^{-1}\cdot\mathbf{D}\right\|}\right)\mathrm{OCR} \tag{11}$$

Figure 4.a shows the Visco-hypoplastic model incorporating the inviscid overconsolidation ratio OCR*. It clearly shows a single isotach for all the given strain rates. Figure 4.b and c show an isotropic compression simulation using Eq. (11) and (12), and finally, Figure 4.c shows a small degradation of *s* when OCR>>1 which experiences an acceleration after yielding.




(a) (b) (c)

Figure 4: a) Inviscid Visco-hypoplastic model, b) Simulations on isotropic compression, c) Structure degradation with the vertical strain.

## SIMULATIONS

The Marl Clay was selected to evaluate the proposed model performance. This clay covers most of the Greek territory and is characterized for showing a different behaviour when using remolded samples. It presents a high cementation which is attributed to carbonates (Anagnostopoulus, et.al. 1990).

The material parameters were calibrated as follows. The compression and swelling index, $\lambda$ and $\kappa$ respectively, were adjusted using the isotropic compression test with the remolded sample. The virgin compression line was selected as the reference isotach and hence parameters $D_r$ and $e_{100}$ may be computed. The viscosity index was computed by using the Leinenkugel's relation (Leinenkugel, 1976):

$$I_v = 0.05 + 0.026 \ln\left(w_L\right) \tag{12}$$

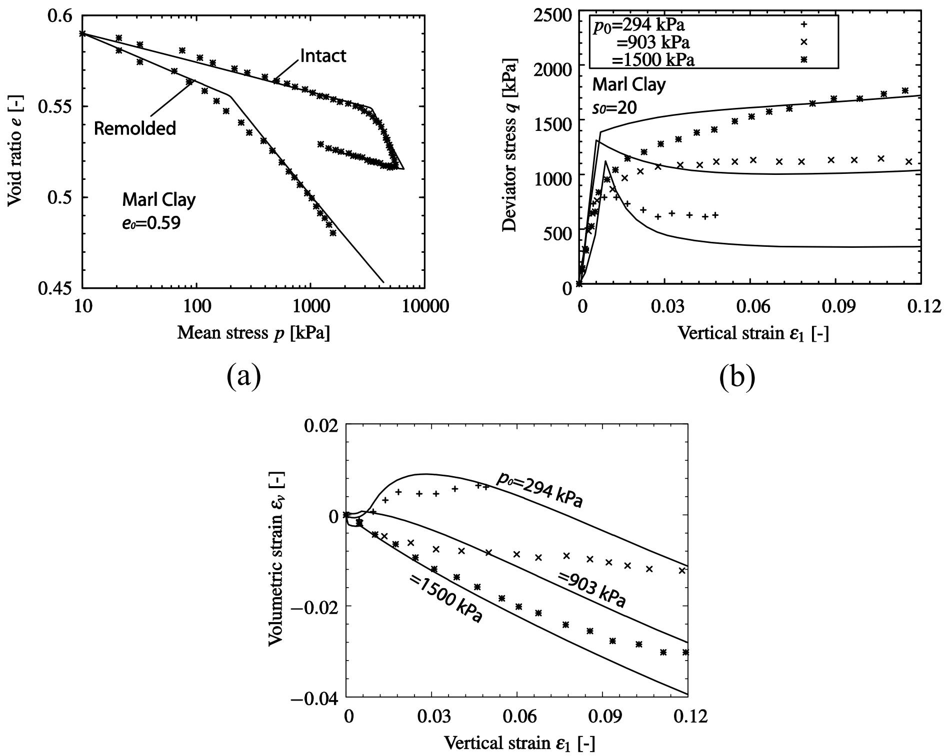
The selected values of the liquid limit $w_L$ and $\varphi_c$ correspond to the values reported on (Anagnostopoulus, et.al. 1990). The values of $s_0, \omega, \Gamma, k$ were calibrated with the intact sample on the isotropic compression test. Finally a typical parameter of $\beta_R$ =0.7 was selected. All the parameters are listed in Table 1 below.

Table 1: Parameters of the Marl Clay

| $e_{100}$ | $\lambda$ | $\kappa$ | $\beta_R$ | $I_v$ | $D_r$ | $\varphi_c$ | $k$ | $\Gamma$ | $\omega$ | $s_0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| [-] | [-] | [-] | [-] | [-] | [s$^{-1}$] | [°] | [-] | [-] | [-] | [-] |
| 0.58 | 0.022 | 0.0073 | 0.7 | 0.018 | 1.0E-6 | 18 | 1.7d1 | 1.0d-3 | 8d-3 | 20 |

Figure 5.a-c show the simulations on the Marl Clay. Figure 5.a show the simulation of the isotropic compression test used for the calibration of paramters. Figure 5.b-c shows the preditcion of a drained triaxial test with the proposed model. Despite that the predictive curves simulate fairly well the experimental curves, there

still some limitations in the model. Apparently, a strong contractant behaviour appears after yielding while the structure still degradating. This behaviour may be improoved in a future work.




(a) (b)

(c)

Figure 5: Simulations on the Marl Clay with proposed model.

**Numerical implementation**

A subroutine UMAT for Abaqus was developed. Following Niemunis (Niemunis, 2003), the numerical instabilities due to the exponent in $\mathbf{D}^{vis}$ may be avoided by expanding this tensor with the Taylor's series:

$$\Delta\mathbf{T} = \mathsf{L}:\Delta\boldsymbol{\varepsilon} - \left(\mathbf{D}^{vis} + \frac{\partial\mathbf{D}^{vis}}{\partial\mathbf{T}}:\Delta\mathbf{T} + \frac{\partial\mathbf{D}^{vis}}{\partial e}(1+e)\mathbf{1}:\Delta\boldsymbol{\varepsilon} + \frac{\partial\mathbf{D}^{vis}}{\partial s}\Delta s\right) \quad (13)$$

Noting that $\Delta\mathbf{T}$ appears in both sides of the equation, it may be rewritten as follows,

$$\Delta\mathbf{T} = \mathsf{K}:\left(\Delta\boldsymbol{\varepsilon} - \mathsf{C}:(\mathbf{D}^{vis} + \mathbf{S})\Delta t\right) \quad (14)$$

where $K$, $C$ are defined as,

$$\mathsf{K} = \left(\mathsf{I} + \mathsf{L}:\mathsf{A}\Delta t\right)^{-1}:\mathsf{L}:(\mathsf{I} - \mathsf{B}\Delta t) \quad (15)$$

$$\mathsf{C} = (\mathsf{I} - \mathsf{B}\Delta t)^{-1} \quad (16)$$

and the second order tensor **S** is defined as,

$$\mathbf{S} = \frac{\partial \mathbf{D}^{vis}}{\partial s} \dot{s} \Delta t = -\frac{D_r}{I_v} \left( \frac{1}{\text{OCR}} \right)^{\frac{1}{I_v}-1} \left( -\frac{1}{s \cdot \text{OCR}} \right) \vec{\mathbf{B}} \dot{s} \Delta t \tag{17}$$

Tensors $A$ and $B$ are defined following (Niemunis, 1996),

$$\text{A} = \mathbf{D}^{\text{vis}} \text{OCR} \frac{1}{I_v p_e} \frac{\partial p_e}{\partial \mathbf{T}}$$

$$\text{B} = \frac{1+e}{1-\lambda} \mathbf{D}^{\text{vis}} \otimes \mathbf{1} \tag{18}$$

Finally, the Jacobian matrix J may be approximated to the fourth order stiffness tensor $K$ ,

$$\text{J} = \frac{\partial \Delta \mathbf{T}}{\partial \Delta \boldsymbol{\varepsilon}} \approx K \tag{19}$$

## CONCLUSIONS

The proposed model is able to reproduce the structure influence on the materials mechanical behavior, and conserves the capability of reproducing the viscous effects as the reference model. The model proposes a structure degradation which depends on the OCR in such a way that the degradation accelerates when the plastic range is reached. Simulations on the Marl Clay which are characterized for presenting a high structure ($s_0$>10) adjust to the experimental curves fairly well. Some improvements may be useful to simulate the dilatancy of these materials. Finally, a User Subroutine UMAT for Abaqus has been developed and satisfactorily implemented.

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